BANDWIDTH ALLOCATION IN NETWORKED CONTROL SYSTEMS VIA NASH BARGAINING

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Abstract— In this paper we propose a wireless networked control system (NCS) bandwidth allocation method based on the classical Nash Bargaining Solution. A NCS in closed-loop architecture is described for simulation purposes. Constraints as time delay, packet loss and transmission dynamics are incorporated to the network channel model. A controller design procedure is also proposed based on the NCS setup needs. Simulation results with the proposed method and the equilitarian bandwidth division are presented. The proposed method presents better performance than the equilitarian method and some others existing dynamic bandwidth allocation methods.

Resumo— Neste artigo é proposto um método de alocação de banda para um sistema de controle em rede (NCS) sem fio com base na solução clássica da Barganha de Nash. Uma NCS de arquitetura em malha fechada é implementada para fins de simulação. Fatores como atraso, perda de pacotes e dinâmica de transmissão são incorporados ao modelo do canal de transmissão da rede. Um procedimento de projeto de controlador também é apresentado com base nas necessidades da NCS implementada. Os resultados da simulação com o método proposto e com a divisão igualitária banda são apresentados. O método proposto apresenta um desempenho melhor do que o método igualitário e alguns outros métodos de alocação dinâmica de banda existentes.

Keywords— Process Control, Networked Control Systems, Bandwidth Allocation, Nash Bargaining.

1 Introduction

Network resources management is an important subject in numerous engineering areas. As any communication network transports a limited amount of information per unit of time (see for example Hespanha et al. (2007)), in many applications this constraint can be the most important for systems operation. Network bandwidth and its allocation are typical tasks that might limit systems performance.

Among the strategies and methods for network bandwidth allocation, Game Theory (Osborne and Rubinstein, 1994) mechanisms such as auctions and pricing have become alternatives. In Yaïche et al. (2000), Game Theory concepts, specifically Nash Bargaining, are used to build up methods in order to maximize network resources utilization. The authors present a concept of rate control, and consider the bandwidth allocation for each node of the network as a control variable. Bandwidth pricing methods and auction mechanisms applied to centralized and decentralized control architectures are used to determine an optimal allocation.

Networked control systems, NCSs, are structures that use one or more shared networks for information exchanging. Network bandwidth allocation is an important topic in NCSs, as sensors, actuators and controllers are distributed among the system, and share the communication network (Hespanha et al., 2007). NCSs are used in applications such as robotics, embedded systems, vehicle electronics and power systems.

Several different architectures apply to NCSs, but the most frequent are the so-called open-loop and closed-loop. In Tipsuwan et al. (2009) an example of open-loop architecture is discussed. The controller, actuator and plant are directly connected, with the reference signal transmitted to the rest of the system via the network. In Senol et al. (2011) and Jungers et al. (2013), closed-loop architectures are used. In the later case, the controller is connected to the actuator through the network. Regardless the case, the network transmission delay can lead the control system to instability.

Bandwidth allocation applied to NCSs is the focus of Tipsuwan et al. (2009). It is proposed an auction-based method that uses dynamic pricing for determining the bandwidth allocation in a dynamic framework. The main design parameters of the method are the systems sensitivities. Similar to Tipsuwan et al. (2009), Yan et al. (2013) also present a bandwidth allocation method based on Game Theory and Mechanism Design, with error approximation functions and minimum bandwidth for each node (player) as main ingredients. However, Yan et al. (2013) uses bandwidth distributions defined a priori and remain constant during the running time, which does not allow the NCS to adapt to node requirements.

In this paper, the Nash Bargaining Solution, NBS, (Osborne and Rubinstein, 1994) provides a cooperative framework for bandwidth allocation in NCSs. It is shown that the NBS represents a
simple and effective method for dynamically allocate network resources as bandwidth. The paper is organized as follows. In Section 2, the bandwidth allocation problem in NCSs is addressed and some performance parameters for future comparisons are introduced. Modeling this problem in a Game Theory framework is the main subject of Section 3, which also presents the NBS dynamic bandwidth allocation method and a standard method for comparison purposes. Section 4 presents some simulation results and Section 5 the main conclusions of the work.

2 Problem Description

Modeling issues for NCSs are the transmission delay, the format in which information is transmitted over the network and the packet loss phenomenon. It is important to characterize the transmission channels as well as the problems and restrictions associated with transmission. This is essential for the applicability of the method to be presented to real NCSs.

The network transmission delay is a sum of several elements, such as scheduling mechanisms, hardware technology, network physical layer and, mainly, bandwidth allocation. In Tipsuwan et al. (2009), Senol et al. (2011) and Yan et al. (2013), the transmission delay is the result of various network processes. In NCSs, the transmission delay becomes a central issue, as emphasized in Jungers et al. (2013), for example.

For this study, the transmission delay is referred only to the time used by the network to send information between nodes. According to Hespanha et al. (2007), other time bottlenecks, such as data processing, buffers, etc., are usually much smaller than the transmission time, and a possible way to model the transmission delay can be a transfer function such as \( D(s) = e^{-\tau s} \), where \( \tau \) is the delay parameter in seconds.

In networks it is common to use pieces of information called packets. The format and size of a packet describe how much information is delivered in a single transmission time event. Any transmission channel, simple or sophisticated, is submitted to failures during information transmission. In digital transmission systems, the bit failure ratio (BER) represents a failure probability during transmission. Those failures may happen due to different phenomena, such as collision and lack of synchronization, among others, and could be used as a stochastic parameter that causes also packet loss.

In Zhang et al. (2013) and Hespanha et al. (2007), the packet loss in NCSs is modelled as a random Bernoulli process \( \theta_i \in \{1, 0\} \), \( \forall i \in \mathbb{N} \), where \( \theta_i = 1 \) means that the signal \( y_i \) sent at time \( i \) successfully reached its destination; \( \theta_i = 0 \), otherwise. Here, the random packet loss process is defined by using BER as \( p = BER := P(\theta_i = 0) \in [0, 1], \forall i \in \mathbb{N} \), where \( p \) is the failure probability during a given transmission.

In this paper we assume that a packet provides information of a given signal at a specific sampling time. The packet structure used is the same suggested by Tipsuwan et al. (2009): The first 14 bytes are the preamble, followed by 2 bytes indicating the packet start, 2 bytes for addressing and 4 bytes for signal data. The packet end has 2-byte Cyclic Redundancy Check, CRC, for error checking. The total packet size is 24 bytes.

In NCSs old or outdated information is not relevant and do not need to be transmitted (Hespanha et al., 2007). According to Zhang et al. (2013), this is the most effective way to avoid problems such as transmission delay and packet loss. The general transmission channel model used in this paper is described in Figure 1. The network architecture adopted is the closed-loop, illustrated in Figure 2.

The NCS model adopted assumes a wireless network implementation. Bandwidth in wireless networks is a much more scarce resource than in wired ones. The wireless router, the component responsible for routing packets and allocating bandwidth, is the link between the network nodes.

The packet traffic on the network is bidirectional due to the closed-loop feedback architecture. The network physical layer must be shared for transmission, but simultaneous transmission in both directions is not possible. This constraint is not always present since several wireless technologies can support multiple transmission channels. However, in order to emphasize the limited network bandwidth issue, this constraint is included in our model.

The typical wireless network transmission rate adopted is 38.400 bits/s, as well as a BER of \( 10^{-5} \). In this study, it is suggested to con-
sider the network delay parameter calculated by
\[ \tau = 8 \times \frac{\text{PacketSize}}{\text{TransmissionRate}}, \]
once it is considered only transmission time for the network delay model.

We assume that the NCS model is centralized, with bandwidth allocation and packet routing determined by the wireless router. In a general sense, the wireless router establishes at every transmission time slot which nodes communicates to others. Executing this procedure over the operation time correspond to the bandwidth allocation of the network.

As suggested in Tipsuwan et al. (2009), a time frame scheme must be defined for the wireless router operation. For every time window, the total operating period \( T_O (0.500s, \text{selected for simulations}) \), the wireless router repeats the following procedure: During the first instants, the polling period \( T_p (0.030s) \), it receives requests from the network nodes. Then, during the allocation period \( T_S (0.005s) \), the wireless router processes all the information and allocates bandwidth to each network node for the remaining time, the transmission period \( T_T (0.465s) \). Another important time period is the sampling period \( T_S (0.005s) \), the time window in which two network nodes, a transmitter and a receiver, are connected through the wireless router and the information is exchanged. \( T_S \) is the smallest time slot used in the bandwidth allocation process.

In NCSs the scheduling algorithm is as important as the bandwidth allocation. As one of our objectives is to compare the efficiencies of different bandwidth allocation methods in NCSs, the scheduling algorithm should not influence the simulation results. An important assumption is that transmission is scheduled only during the \( T_T \) period. In this paper, a regular round-robin algorithm is used, with an adjustment due to the closed-loop architecture: two consecutive transmission slots are scheduled in order to have a control signal transmitted and a feedback signal received.

As most traditional control systems, NCSs involve reference signals, controllers, actuators and plants. We are particularly interested in controlling DC motors (Maxon A-max 226802). Applications in industry that use DC motors are common, such as in positioning and conveyors speed control. The process variable to be controlled is the angular speed, \( \Omega \); \( \Omega \) is the reference command for \( \Omega \).

Actuators are devices that receive the signal from the controller and convert it into the plant input signal. The plant transfer function is Franklin et al. (1997).

\[ P(s) = \frac{\Omega(s)}{V(s)} = \frac{k_t}{(Js + b)(Ls + R) + k_c k_t}, \]

where \( V(s) \) is the voltage applied to the motor terminals (actuating signal), \( k_t \) is the motor gain, \( J \) is the equivalent inertia, \( b \) is the equivalent viscous friction, \( L \) is the inductance, \( R \) is the resistance of the electric circuit and \( k_c \) is the back electromotive force of the (armature controlled) motor. The actuator transfer function is

\[ A(s) = \frac{P(0)^{-1}}{cs + d} = \frac{bR + k_c k_t}{k_t(cs + d)}, \]

The DC motor parameters used in this paper are those provided in Maxon Motor (2013). A proportional-integral-derivative controller with derivative filter was adopted. The controller transfer function is

\[ G(s) = \frac{K_P + \frac{K_I}{s}}{1 + \frac{K_D}{s}}, \]

where \( K_P \), \( K_I \) and \( K_D \) are the controller gains and \( p \) is the pole of the filter.

A performance indicator for control systems is the error between the reference signal \( r(t) \) and the plant output \( y(t) \), \( e(t) = r(t) - y(t) \). The tracking error is the basic performance index for comparison of control strategies.

A more elaborated index is the integrated absolute error, IAE, calculated over every integration period \( T_O \), defined as

\[ \bar{V}(t) = \frac{\int_{t}^{t+T_O} |r(\tau) - y(\tau)| \, d\tau}{T_O}. \]

We consider \( N \) plants (DC motors) to be controlled by the NCS (Figure 2). Each plant corresponds to an agent (player) in a Game Theory setting. The overall performance index \( Q \) is given by the average \( \bar{V} \) over the simulation period \( T_{total} \)

\[ Q = \frac{\int_{0}^{T_{total}} \left( \sum_{k} \bar{V}_k(t) \right) \, dt}{T_{total}}, \]

where \( \bar{V}_k \) is the IAE index of agent \( k \) over the total simulation period horizon. The higher \( Q \), the worse the overall system performance.

3 Bandwidth Allocation Methods

In this section we formally set the NCS bandwidth allocation problem as a game. Each individual control system \( k \) (reference signal, controller, actuator and plant) is considered a player; \( N \) is the number of players in the NCS bandwidth allocation game. Players compete for a limited bandwidth and the game is repeated every \( T_O \) seconds. After \( T_A \), every player \( k \) receives a percentage of the total available bandwidth, \( x_k \geq 0 \);

\[ x = [x_1, x_2, ..., x_k, ..., x_{N-1}, x_N] \]

is the game outcome and \( \sum_{k=1}^{N} x_k \leq 1 \).
When control systems are implemented via a network, the sampling rate is an important issue. According to Franklin et al. (1997), the selection of the sampling rate for a digital control system must consider a trade off. In general, the higher the rate, the better the system performance. However, higher sampling rates usually imply in higher costs. It is important to know the minimum operational rate for the system and how it affects the overall system performance. The minimum sampling rate theorem indicates that the sampling rate \( \omega_s \) of a control system must be at least twice as its closed-loop bandwidth \( \omega_b \), or \( \omega_s > 2\omega_b \) (Franklin et al., 1997). The minimum sampling rate theorem provides a minimum operational limit, but it does not guarantee good overall system performance. In digital control systems design, the sampling rate should be at least about twenty times faster than system bandwidth, or \( \omega_s \geq 20\omega_b \) (Franklin et al., 1997).

In this paper, the transmission period \( T_T \) (465ms) is the time window for exchanging information between set point, controller, actuator and feedback, while the network sampling period is \( T_N \) (5ms) which gives a sampling rate of 200Hz for the whole network. This is the maximum sampling rate considering that 100% of the network bandwidth is available. If only a percentage of the total available bandwidth is allocated to a given control system, its real sampling rate diminishes at the same percentage.

### 3.1 Controller Design

The controller design was developed using Matlab/Simulink/Control System Toolbox. The overall performance specification is to obtain a small settling time, with similar characteristics to critical damping, while keeping system bandwidth lower than 20% of the total available network bandwidth (40Hz). The control system parameters \( c, d, K_p, K_I, K_P \) and \( p \) used in this simulations are given by 5, 1, 164.3, 44.3, -0.7 and 30.8. The closed-loop bandwidth of each control system is \( \omega_b = 6.055 \)Hz. As in Yan et al. (2013), a minimum network bandwidth \( x_{min} \) for each control system \( k \) was established.

Since in the closed-loop architecture both the control and the feedback signal are transmitted using the network, it is necessary to assume that the minimum sampling rate is capable of providing a complete information exchange cycle. This means that the minimal sampling rate should be enough for transmitting a control signal, receiving the sensor feedback, sending the adjusted control signal and receiving the measured output by feedback.

Considering the minimum sampling rate theorem, the minimal sampling rate for each control system must be at least 8 times the closed-loop bandwidth. The minimum bandwidth \( x_{min} \) considered for each control system designed in this paper is 0.2422.

### 3.2 Bandwidth Allocation via NBS

A bargaining game involves \( N \) players and a subset of possible outcomes, represented as \( N \)-dimensional coordinate points, one of which must be selected as the solution. In this paper, the outcomes are the images of the utility functions \( u_k \) of \( N \) players, and the point selected - Nash Bargaining Solution, or NBS - satisfies four axioms discussed in details in (Osborne and Rubinstein, 1994).

The disagreement point in the NCS bandwidth allocation game is \( d_k = 0 \) for each \( k \), which means that the bandwidth is not distributed among the players. In this case, the objective function of the Nash Bargaining problem is \( \prod_{k=1}^{N} u_k(x_k) \), where \( u_k(x_k) = 1 + x_k \cdot e_{ITAE} \) if \( x_k \geq 0 \), and 0, otherwise, where \( e_{ITAE} \) is the integrated absolute error multiplied by the time for player \( k \):

\[
e_{ITAE} = \int_0^T t \cdot e_k(t) \ dt.
\]

Our definition of \( u_k(x_k) \) stresses the fact that the utility of \( x_k \) increases when \( e_{ITAE} \) increases.

The NBS is the optimal solution of the problem

\[
\max_x \prod_{k=1}^{N} u_k(x_k) \\
\text{s.a. } \sum_{k=1}^{N} x_k \leq 1 \\
x_{min} \leq x_k \leq 1, \quad k = 1, 2, ..., N.
\]

Problem (1) is equivalent to the convex optimization problem

\[
\max_x \sum_{k=1}^{N} \ln(u_k(x_k)) \\
\text{s.a. } \sum_{k=1}^{N} x_k \leq 1 \\
x_{min} \leq x_k \leq 1, \quad k = 1, 2, ..., N.
\]

Using the linear transformation \( x_k = x_{min} + x_k' \), problem (2) can be rewritten as

\[
\max_{x'} \sum_{k=1}^{N} \ln \left( 1 + x_k' \cdot e_{ITAE} \right) \\
\text{s.a. } \sum_{k=1}^{N} x_k' \leq (1 - \zeta) \\
x_k' \geq 0, \quad k = 1, 2, ..., N.
\]

\[
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Using the linear transformation } x_k = x_{min} + x_k', \text{ problem (2) can be rewritten as

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\max_{x'} \sum_{k=1}^{N} \ln \left( 1 + x_k' \cdot e_{ITAE} \right) \\
\text{s.a. } \sum_{k=1}^{N} x_k' \leq (1 - \zeta) \\
x_k' \geq 0, \quad k = 1, 2, ..., N.
\]
where $\varsigma = \sum_{k=1}^{N} x_{\min k}$.

The Lagrangian of (3) is

$$L(x', \lambda) = \sum_{k=1}^{N} \ln \left( 1 + x_k' \cdot e_{ITAE}^k \right) + \lambda_0 \left( 1 - \varsigma - \sum_{k=1}^{N} x_k' \right) + \sum_{k=1}^{N} \lambda_k x_k', \quad (4)$$

and considering the Karush-Kuhn-Tucker condition $\partial L(x_k', \lambda)/\partial x_k' = 0$, we obtain

$$x_k'^* + \frac{1}{e_{ITAE}^k} = \frac{1}{(\lambda_0^* - \lambda_k^*)}, \quad k = 1, 2, ..., N.$$

If $x_k'^* > 0$, then $\lambda_k^* = 0$ and

$$x_k'^* = \frac{1}{\lambda_0^*} - \frac{1}{e_{ITAE}^k}, \quad k = 1, 2, ..., N.$$

Considering the constraint $\sum_{k=1}^{N} x_k' \leq (1 - \varsigma)$, the optimal multiplier $\lambda_0^*$ must satisfy

$$\frac{1}{\lambda_0^*} = \frac{1}{A} \left( 1 - \varsigma \right) \sum_{k=1}^{N} \left( \frac{1}{e_{ITAE}^k} \right)$$

where $A$ is the number of nonzero $x_k'^*$s.

3.3 Equalitarian Bandwidth Allocation

The method chosen for comparison purposes was the equalitarian bandwidth division. This is a simple and intuitive way to allocate bandwidth for all players, and one of most effective methods tested for the specific setup in this paper. The method establishes that $x_k = 1/N$, for $k = 1, 2, ..., N$. This method is static, that is, it does not consider the dynamic information about the NCS state.

4 Results

For simulation purposes, we considered sinusoidal reference signals $r_k$ described by $2 + \sin(\alpha_k t/T_S)$, with different frequencies $\alpha_k T_S$, where $T_S$ is the NCS sampling period. The values of $\alpha_k$ for each player $k (k = 1, 2, 3)$ were 0.1, 0.01 and 0.001.

Matlab R2013a and its extension Simulink were chosen as simulation platform for the experiments. The schematic diagram of the simulation setup is shown in Figure 3.

Due to space limitations, only the results for player 1, the one with the highest $\alpha_k$ and hence with the higher network resource demand in the NCS are presented.

Figure 4: Error and control signals of Player 1.

In Figure 4, the average error amplitude is lower with the proposed method. In addition, the error signal generated by the NBS method is confined to a smaller region, while the equalitarian method generated higher error peaks. A similar behavior is observed in the control signals. The control effort spent by the proposed method is significantly smaller.

Figure 5 shows the relationship between the performance parameter $V_1$ and the allocated...
bandwidth for player 1 during the simulation time when the NBS method is used. After the first operation period $T_D$, the value of $\bar{V}_1$ rises quickly. The NCS response is to allocate to player 1 a larger bandwidth percentage. Then, a drastic decrease in $\bar{V}_1$ is verified, followed by a smaller bandwidth percentage allocated to player 1. Increase in $\bar{V}_1$ leads to an increase in bandwidth, and then a certain stability in terms of $\bar{V}_1$ and bandwidth is achieved.

It is interesting to compare the values of $e_{ITAE_1}$ provided by both methods at the end of the experiment. The equalitarian method provided $e_{ITAE_1} = 27.3$, and the proposed method $e_{ITAE_1} = 18.7$, an improvement of 31.6%, which reflects a better bandwidth distribution according to the needs of player 1. Other dynamic bandwidth allocation methods (Tipsuwan et al., 2009), (Yan et al., 2013) performed worse when compared to the equalitarian method under the present simulation conditions.

Considering the overall system performance of all players, the equalitarian method produced $\bar{Q} = 0.66392$, and the proposed method $\bar{Q} = 0.65722$. The proposed method performs slightly better. Notice that $\bar{Q}$ does not weight absolute errors with their temporal occurrences. We observe that the equalitarian method had provided the best result for the adopted simulation setup when compared to other dynamic bandwidth allocation methods.

5 Conclusion

The results obtained in this paper show that the classical Nash Bargaining Solution applied to the NCS bandwidth allocation problem can bring efficiency to such systems.

The method proposed in this paper performed better than the standard equalitarian bandwidth method. It improves the performance of players with higher bandwidth demands due to its dynamic character. The overall system performance is also improved, which is consistent with the cooperative nature of the NBS.

In a future work, we will develop an accurate comparison of our method with other dynamic bandwidth allocation methods. Alternative implementations of the NBS in other architectures and setups are also issues to be considered in future works.

References


