UNSCENTED KALMAN FILTER FOR ATTITUDE ESTIMATION OF SATELLITES

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Abstract—In this work, a new Unscen ted Kalman Filter (UKF) composed by the only consistent minimum sigma-representation in the literature is proposed for the problem of estimating the attitude of a satellite. It is supposed that the system is equipped with a three-axis magnetometer (TAM) and gyroscope rate sensors. Since the system is modeled using unit quaternions and the standard Unscen ted Kalman Filter uses operations of sums, this filter can not be applied straightforwardly to the system considered here. We, then, propose a solution using rotation vectors, an euclidean parameterization of the unit quaternion set. In numerical simulations, the new filter is shown to provide good estimation quality and also to outperform the Multiplicative Extended Kalman Filter.

Keywords—Unscen ted Kalman Filter, Stochastic Estimation, Attitude Estimation.

1 Introduction

The attitude of a satellite can be obtained by using a tree-axis magnetometer (TAM) and gyroscope rate sensors (Crassidis et al., 2007). Nonetheless, since there always are uncertainties in the considered model for the system and the measurements are noisy, estimators, such as Kalman Filters, have to be used in order to obtain reliable estimates of the attitude of the considered satellite. For non-linear systems, like the one considered here, non-linear filters should be used, which is the case of the Unscen ted Kalman Filter (UKF).

The UKF has been object of a wide brand of research in the literature since its first proposal. Real applications varies from unmanned aerial vehicle’s attitude estimation (Marina et al., 2011) and target tracking (Macagiano and Freitas de Abreu, 2012) to estimation of the states of polymer electrolyte membrane fuel cell (Vepa, 2012), among others. In relation to the widely known Extended Kalman Filter (EKF), this filter has the advantage of providing a second order approximation of the estimate – the conditional mean – , while the EKF provides only a first order one. Besides, the UKF is derivative free, while the EKF must calculate the Jacobian at each time step. These filters are solutions to the problem of estimation of (real) stochastic dynamic systems, which can be in the additive form

\[ x_k = f(x_{k-1}, k) + q_k, \quad y_k = h(x_k, k) + r_k, \]  

or in the more general form

\[ x_k = f(x_{k-1}, q_k, k), \quad y_k = h(x_k, r_k, k), \]  

where \( k \) is the time step, \( x_k \in \mathbb{R}^n \) is the internal state, \( y_k \in \mathbb{R}^m \) is the measured output, and \( q_k \in \mathbb{R}^n \) and \( r_k \in \mathbb{R}^m \) are the process and measurement noises, respectively.

The UKF is a recursive application, in a Kalman Filter prediction-correction framework, of the so called Unscen ted Transform (UT), which, in turn, can be seen as an approximation of a given transform, say \( Y = f(X) \), by two sets of weighted points deterministically chosen (the sigma points) which approximate the joint probability density function of \( X \) and \( Y \) via moment matching (Särkkä, 2007).

However, applying the UKF to the system considered here can not be done directly since the attitude of the satellite is represented with a unit quaternion and sums of unit quaternions (which occur in the UKF) do not result, generally, in a unit quaternion. Unit quaternions are important in applications in which some kind of rotation is considered because every element of \( S^3 \) (the sphere of dimension 3, which is the set of unit quaternions) can be associated with an element of the \( SO(3) \), the special group of orthogonal matrices which are the ones that represent rotations (Altmann, 1986), with the advantage over other parameterizations of the \( SO(3) \) of not having any singularity. Quaternions are represented in this work in bolded font (ex. \( \mathbf{x} \)).
In order to circumvent this problem, the Unscented Kalman Filter for attitude estimation of satellite proposed here is composed by rotation vectors, a parameterization of the $S^3$, in the steps of the UKF that have operations of sums. Another improvement of the new UKF over the other ones for attitude estimation of satellite is that it is constituted by the only consistent minimum set of sigma points in the literature (cf. (Menegaz et al., 2014)). In numerical simulations, this new UKF is shown to provide good estimates and also to outperform the Multiplicative Extended Kalman Filter (one of the most used filters for the problem considered here, cf. (Crassidis et al., 2007)].

Following this introduction, Section 2 presents an introduction to the quaternion algebra and states the system modeling the attitude of the satellite, and Section 3 briefly reviews the theory of Unscented Kalman Filtering. Afterwards, Section 4 proposes the new Unscented Kalman Filter for attitude estimation of satellites. Section 5 presents a numerical example and Section 6 provides the conclusions of this paper.

Throughout this work, the following notations are used:

- For a matrix $A$, $\sqrt{A}$ stands for a square-root matrix of $A$ such that $A = \sqrt{A}^T \sqrt{A}$ for its ith row and jth column element; $A_{i,j}$ and $A_{j,i}$, respectively, for its jth column and ith row; and $(A|\psi)^T$ for $(A^T|A)$ for a block matrix consisting on $A$ being repeat $r$ times on the rows and $q$ on the columns.

- For a set $X = \{x_i, w_i\}_{i=1}^N$, $\mu_X := \sum_{i=1}^N w_i x_i$ is its sample mean and $\Sigma_X := \sum_{i=1}^N w_i (x_i - \mu_X)^T (x_i - \mu_X)$ its sample covariance.

Henceforth, consider a random variable $X \sim (\bar{X}, P_{XX})$, a function $f : \mathbb{R}^n \to \mathbb{R}^m$ defining $Y := f(X)$ such that $Y \sim (\bar{Y}, P_{YY})$, the cross-covariance matrix $P_{XY}$ of $[X, Y]^T$, and the sets $X = \{x_i, w_i\}_{i=1}^N$ and $Y = \{y_i, w_i\}_{i=1}^N$ of $\gamma = \{\gamma_i, w_i\}_{i=1}^N$. Along the text, we suppose that $q_k \sim N([0]_{n \times 1}, Q_k)$ and $r_k \sim N([0]_{n \times 1}, R_k)$ are uncorrelated.

2 Attitude Estimation of Satellites

An attitude estimation problem of satellites can be modeled using quaternions (cf. (Crassidis et al., 2007)]. Quaternions form a four-dimensional algebra over the real numbers and can be used to parametrize the $SO(3)$ (Cohn, 2003). By the fact that “globally nonsingular threedimensional parameterization of the rotation group is topologically impossible” (Markley, 2003), they are a good choice to represent rotations in comparison to other three dimension parameterizations, such as the Euler angles. In fact, a unit quaternion can represent a rotation without any singularity (Markley, 2003).

The algebra of quaternions, denoted by $\mathbb{H}$, is generated by its basis elements, 1, $i$, $j$ and $k$, whose multiplication is defined pairwise as (Cohn, 2003):

$$-ji = jk = \bar{k}i = i, \quad -jk = ji = \bar{i}k = j.$$  

Hence, an element of $\mathbb{H}$ is of the form $q := q_1 + iq_2 + j q_3 + k q_4$, where $q_1, q_2, q_3, q_4 \in \mathbb{R}$ are called the Euler symmetric parameters or the Euler-Rodrigues parameters (Markley, 2003).

The sum (subtraction) of two quaternions $a = a_1 + ia_2 + ja_3 + ka_4$ and $b = b_1 + ib_2 + jb_3 + kb_4$ is defined by the sum (subtraction) of its basis elements: $a \pm b := a_1 \pm b_1 + i (a_2 \pm b_2) + j (a_3 \pm b_3) + k (a_4 \pm b_4)$. The multiplication is also defined by its basis elements:

$$a \circ b := \left(a_1 a_4 + a_2 a_3 + a_3 a_4 + a_4 a_1\right) \times \left(b_1 + ib_2 + jb_3 + kb_4\right)$$

$$= (a_1 b_1 - a_2 b_4 - a_3 b_3 - a_4 b_2) + i(a_1 b_2 + b_1 a_2 + a_3 b_1 - a_4 b_3) + j(a_1 b_3 - b_1 a_3 + a_4 b_1 + a_3 b_2) + k(a_1 b_4 + a_4 b_1 - a_3 b_2 + a_2 b_3).$$ (3)

For a quaternion $q$, the element $q^{-1} \in \mathbb{H}$ is its inverse if $q \circ q^{-1} = q^{-1} \circ q = 1$. In analogy with complex numbers, the norm and the conjugate of $q$ are defined in order to calculate its inverse of an arbitrary quaternion. The conjugate of a quaternion $q$, $q^* \in \mathbb{H}$, is $q^* := \text{Re}(q) - i \text{Im}(q)$ and the norm is $||q|| := \sqrt{\text{Re}^2(q) + \text{Im}^2(q)}$. In this way, if $||q|| \neq 0$, the inverse is $q^{-1} := \frac{q^*}{||q||^2}$.

If $||q|| = 1$, we call $q$ a unit quaternion or quaternion of rotation. The set of unit quaternions form a group under the quaternion multiplication defined in (3), but not under the sum (Altmann, 1986), which hampers the creation of...
UKF’s for quaternion systems. For every rotation \( R(n, \theta) \), where \( \|n\| = 1 \), \( n \) is the axis of rotation and \( \theta \in \mathbb{R} \) is the angle of rotation, there are two associated unit quaternions \( q \) and \( q' \) such that \( q = \cos \left( \frac{\theta}{2} \right) + n \sin \left( \frac{\theta}{2} \right) \), \( q' = -q \). (Cohn, 2003). Therefore the \( SO(3) \) can be parametrized by unit quaternions, but the set of all unit quaternions covers the \( SO(3) \) twice.

The attitude of a satellite can be described by the following differential equation:

\[
e(t) = \frac{1}{2} T_m u(t) \otimes e(t) (4)
\]

where \( e(t) \in S^3 \) is the attitude of the satellite and \( u(t) \in \mathbb{R} \) is a control input. We supposed that the satellite is equipped with a tree-axis magnetometer (TAM) and gyroscopic rate sensors. It is assumed that corrupted measurements \( \tilde{u}(t) \) of the angular velocity \( u(t) \) are provided by biased gyros

\[
\tilde{u}(t) = u(t) + \beta(t) + w_n. \quad (5)
\]

where \( T \) is the discretization step, \( w_n \sim N \left[ 0, 0, 0 \right]^{T}, \sigma^2_N I_3 \) is a zero mean Gaussian noise, \( \sigma_u \) is the standard deviation of the gyro measurements, and \( \beta(t) \in \mathbb{R}^3 \) is drift error with \( \beta(t) = w_o(t) \sim \left[ 0, 0, 0 \right]^{T}, \sigma^2_N I_3 \).

Acquired measurements are modeled by the following equation, for \( i = 1, 2, 3 \),

\[
y^{[i]}_k = C_k d^{[i]} + v^{[i]}_k, \quad (6)
\]

where

\[
C_k = \begin{bmatrix}
 x_1^2, k & x_2^2, k & 2 x_1 x_2, k \\
 2 (x_2, k x_3, k) & x_1^2, k & x_2^2, k \\
 2 (x_2, k x_4, k) & x_1 x_2, k & 2 x_3 x_4, k \\
 2 (x_1, k x_3, k) & x_1^2, k & 2 x_2^2, k \\
 2 (x_1, k x_4, k) & x_1 x_2, k & 2 x_3 x_4, k \\
 2 (x_2, k x_3, k) & x_1 x_2, k & 2 x_3 x_4, k \\
 2 (x_1, k x_3, k) & x_1^2, k & 2 x_2^2, k \\
 2 (x_1, k x_4, k) & x_1 x_2, k & 2 x_3 x_4, k
\end{bmatrix},
\]

\( d^{[i]} \) is a reference direction vector to a known point and \( v^{[i]}_k \) the measurement noise (Crassidis et al., 2007; Crassidis and Markley, 2003). We assume that two directions are available from the TAM and, hence we can have \( d^{[1]} = [1, 0, 0]^{T}, d^{[2]} = [0, 1, 0]^{T} \).

### 3 Unscented Kalman filtering

There are three main concepts that are needed in order to properly understand the Unscented Kalman Filter theory, to name, the ones of a \( \sigma \)-representation, an Unscented Transform and an Unscented Kalman Filter (Menegaz et al., 2015).

A sigma (\( \sigma \))-representation is an approximation of a random variable by a set of weighted points via moment matching, and the Unscented Transform consists of two sets of weighted points (the sigma points) that approximate a joint distribution of two random variables in the case where there is a functional dependence between them. Precisely, the set \( \chi = \{x_i, w_i\}_{i=1}^{N} \) is a \( \sigma \)-representation of \( X \sim (\bar{X}, P_{XX}) \) if

\[
\mu_r = \bar{X} \quad \text{and} \quad \Sigma_{r} = P_{XX} (Menegaz et al., 2015); \quad \text{and, for} \quad \gamma = \{\gamma_i, w_i\}_{i=1}^{N}, \quad \text{if} \ \chi \quad \text{is a} \ \sigma \text{-representation of} \ X, \ \text{then the Unscented Transform is defined by} \ UT(f, \bar{X}, P_{XX}) := [\mu_{\gamma}, \Sigma_{\gamma}, \Sigma_{\gamma, \gamma}]. \quad \text{The UT has interesting properties concerning the estimation of} \ \bar{Y}, \ P_{YY} \ \text{and} \ P_{XY} (Y|X), \ \text{for} \ Y := f(X), \ \text{stands for the} \ Y \ 's \ \text{Taylor Series around} \ \bar{c} \ \text{truncated at the} \ 0 \ \text{th term:}
\]

\[
\mu_{\gamma}^{[X, \gamma]} = \bar{y}^{[X, 2]}, \quad \Sigma_{\gamma}^{[X, 1]} = P_{YY}^{[X, 1]}, \quad \Sigma_{\gamma}^{[X, 1]} = P_{XY}^{[X, 1]}.
\]

Properties like these make the UT a good choice to be used in stochastic filters. Indeed, the UT can be applied in Kalman Filter prediction-correction frameworks to form Unscented Kalman Filters. (Menegaz et al., 2014) proposed a \( \sigma \)-representation that is shown to be the only consistent minimum one (it is composed by \( n + 1 \) sigma points, which is the minimum amount). For \( X \sim (\bar{X}, P_{XX}) \) and \( v := [v_1, ..., v_n]^{T} \in \mathbb{R}^n \), this minimum \( \sigma \)-representation is given by the following equations:

\[
w_{n+1} = \frac{1}{1 + \sum_{i=1}^{n} v_i^2}, \quad (8)
\]

\[
w = w_{n+1}[v_1^2, ..., v_n^2]^{T}, \quad (9)
\]

\[
E := \sqrt{\frac{P_{XX}}{w_{n+1}} (I + w_{n} v^{2})} \ \text{diag}(v)^{-1}, \quad (10)
\]

\[
e := - \frac{1}{w_{n+1}} E w, \quad (11)
\]

\[
[x_1, ..., x_{n+1}] = [E, e] + \left[ \bar{X} \right]_{1 \times (n+1)}.
\]

The parameter vector \( v \) is a tuning parameter with \( n \) degrees of freedom \( (v_i, \ i = 1, ..., n) \) and has the only restriction of not containing a zero element and can, therefore, also have negative values. The function \( MinSR(\bar{X}, P_{XX}) := \{x_i, w_i\} \) maps a random variable \( X \sim (\bar{X}, P_{XX}) \) to the minimum \( \sigma \)-representation of (Menegaz et al., 2014).

### 4 Unscented Kalman Filter for Attitude Estimation of Satellites

The state of the system described in Section 2 belongs to the set \( S^3 \). Some equations within the algorithm of the UKF contain operations of sums and substractions which, in the case of unit quaternions, would not, necessarily, conserve the unity of the norm. Therefore, we can not apply the UKF
for the attitude estimation of satellites as described in Section 2 straightforwardly. To solve the problem of summing unit quaternions, we use the \(\mathbb{R}^3\) parametrization of the \(S^3\) by rotation vectors.

The function \(RQ(x) := x^\theta\) maps a unit quaternion \(x\) to its rotation vector parameterization \(x^\theta\) (we use the notation \(x^\theta\) for a rotation vector parameterization of the unit quaternion \(x\)) where \(\theta := 2 \arccos(|x|)\). Reversely, the function \(QR(x^\theta) := x\) provides the inverse mapping, where \(\theta := ||x^\theta||\), \(x := \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right) x^\theta\).

The UKF for the attitude estimation of satellites is, then, given by the following algorithm:

1. Initialization at time step \(k\):
   \(\mathbf{x}_{k-1|k-1}, \mathbf{P}_{x|x^\theta}_{k-1|k-1}, \mathbf{Q}^k_{k} \) and \(R_k\) are given. Choose \(\alpha \in [0, 1]\).
2. State’s prediction. For \(i = 1, \ldots, N:\)
   \[\left\{ \begin{array}{l}
   \mathbf{x}^i_{*, k-1 | k-1}, w^i \\
   \mathbf{X}^i_{*, k-1 | k-1} = RQ \left( \mathbf{x}^i_{*, k-1 | k-1} \right), \\
   \mathbf{X}^i_{*, k-1 | k-1} = \mathbf{X}^i_{*, k-1 | k-1} \odot \mathbf{x}_{k-1|k-1}, \\
   \mathbf{X}^i_{*, k-1 | k-1} = f \left( \mathbf{X}^i_{*, k-1 | k-1}, \mathbf{k} \right), \\
   \mathbf{x}^i_{k|k-1} = RQ \left( \mathbf{X}^i_{*, k-1 | k-1} \right), \\
   \mathbf{x}^i_{*, k | k-1} = Q^k_{k} \mathbf{x}^{\mathbf{0}}_{*, k-1 | k-1}, \\
   \mathbf{P}^{\mathbf{0}}_{x|x^\theta}_{k|k-1} = \sum_{i=1}^{N} w^i \left( \mathbf{x}^i_{*, k-1 | k-1} - \mathbf{z}^i_{k|k-1} \right) (\mathbf{\phi})^T,
\end{array} \right.\]
3. State’s correction. For \(i = 1, \ldots, N:\)
   \[\left\{ \begin{array}{l}
   \mathbf{x}^i_{*, k | k}, w^i \\
   \mathbf{X}^i_{*, k | k} = RQ \left( \mathbf{x}^i_{*, k | k}\right), \\
   \mathbf{X}^i_{*, k | k} = \mathbf{X}^i_{*, k | k} \odot \mathbf{x}_{k|k-1}, \\
   \mathbf{X}^i_{*, k | k} = \mathbf{h} \left( \mathbf{X}^i_{*, k | k}, \mathbf{k} \right), \\
   \mathbf{y}^i_{k|k-1} = \sum_{i=1}^{N} w^i \gamma^i_{k|k-1}, \\
   \mathbf{P}^{\mathbf{y}}_{y^y|y^y}_{k|k-1} = \sum_{i=1}^{N} w^i \left( \gamma^i_{k|k-1} - \mathbf{y}^i_{k|k-1} \right) (\mathbf{\phi})^T,
\end{array} \right.\]
4. Correction.
   \[\mathbf{K}_k = \mathbf{P}^{\mathbf{y}_y}_{y^y|y^y}_{k|k-1} \mathbf{P}^{\mathbf{y}_y}_{y^y|y^y}_{k|k-1}^{-1}, \]
   \[\mathbf{\tilde{x}}^i_{k|k} = \mathbf{x}^i_{k|k-1} + \mathbf{K}_k (\mathbf{y}_k - \mathbf{y}^i_{k|k-1}), \]
   \[\mathbf{x}_k = RQ \left( \mathbf{\tilde{x}}^i_{k|k} \right), \]
   \[\mathbf{P}^{\mathbf{x}_x}_{x|x^\theta}_{k|k} = \mathbf{P}^{\mathbf{x}_x}_{x|x^\theta}_{k|k-1} - \mathbf{K}_k \mathbf{P}^{\mathbf{y}_y}_{y^y|y^y}_{k|k-1} \mathbf{K}_k^T.\]

## 5 Numerical Example

In this section, we compare numerically new Unscented Kalman Filter for attitude estimation of satellites with the multiplicative Extended Kalman Filter (Crasidis et al., 2007). The attitude history of the satellite is generated by a Runge-Kutta integration (function \textit{ode45} of Matlab) of (4) with

\[
\begin{array}{c}
   \mathbf{u}(t) := \left[ \begin{array}{c}
   p(t) \\
   q(t) \\
   r(t)
\end{array} \right] := \left[ \begin{array}{c}
   0.03 \sin \left( \frac{2 \pi}{500} t \right) \\
   0.03 \sin \left( \frac{2 \pi}{500} t - 300 \right) \\
   0.03 \sin \left( \frac{2 \pi}{500} t - 600 \right)
\end{array} \right],
\end{array}
\]

For the filtering process, it is assumed that corrupted measurement are provided by \(5_1\) with \(T = 0.1s, \sigma_u = \sqrt{10^{-\sigma_u}} \text{rad} \times s^{-1/2}, \sigma_\beta = 10^{-\sigma_\beta} \mu\text{rad} \times s^{-1/2} \text{and} \beta(t) = [0.001, -0.001, 0.0005]^T \text{rad} \times s^{-1}\). The filter’s state at time step \(k\) is \(\mathbf{x}_k := (e(k), \beta_k)\). The process function is (Crasidis and Jenkins, 2012)

\[
\begin{array}{c}
   \mathbf{a}(e(k), \beta_k) = A_k \mathbf{a}(e(k-1), \beta_{k-1}) + w_k,
\end{array}
\]

where \(\mathbf{a}(\cdot) := [e_1, e_2, e_3, e_4]^T \in \mathbb{R}^4\) is the \(\mathbb{R}^4\) vector representation of \(e \in S^3\),

\[
\begin{array}{c}
   \mathbf{s}_k := \mathbf{a}(k) - \beta_k, \quad \mathbf{m}_k := \sin \left( \frac{T}{2} ||s_k|| \right) \frac{s_k}{||s_k||},
\end{array}
\]

\[
A_k := \left[ \begin{array}{c}
   \cos \left( \frac{T}{2} ||s_k|| \right) \mathbf{w}_s^T \\
   \mathbf{w}_s^T
\end{array} \right],
\]

and \(w \sim \mathcal{N} \left( [0, 0, 0, 0, 0]^T, Q \right)\) is the process noise. The measurement function is given by (6) with \(\sigma_v = 10^{-2} \text{T}\). The initial conditions for the filter are \(e(0) = 1, \beta_0 = [0, 0, 0]^T\) and

\[
\begin{array}{c}
   \mathbf{P}_{x|x^\theta}\mathbf{0}^0 = \left[ \begin{array}{c}
   0.5 \mathbf{I}_4 \\
   [0]_{3 \times 3} \mathbf{0}
\end{array} \right],
\end{array}
\]

Figures 1, 2, 3 and 4 compare the correct values of \(e_1, e_2, e_3, e_4\), respectively, with their estimates from the New UKF and the MEKF; and Figures 5, 6, 7 and 8 provides the comparison of their correct values with only the estimates of the New UKF (this is done in order to improve the visualization of the estimates of the New UKF since the ones of the MEKF makes it difficult in the Figures 1-4). From Figures 5 to 8 we can state that the new UKF provides very good estimates for \(e\), and from Figures 1 to 4 we can note the the new UKF outperforms the MEKF sharply.
Figura 1: Values of $e_1$ for both the New UKF and the MEKF.

Figura 2: Values of $e_2$ for both the New UKF and the MEKF.

Figura 3: Values of $e_3$ for both the New UKF and the MEKF.

Figura 4: Values of $e_4$ for both the New UKF and the MEKF.

Figura 5: Values of $e_1$ only for the New UKF.

Figura 6: Values of $e_2$ only for the New UKF.
6 Conclusions

In this work a new Unscented Kalman Filter composed by the only consistent minimum \(\sigma\)-representation in the literature is proposed for the problem of estimating the attitude of a satellite. It is supposed that the system is equipped with a tree-axis magnetometer (TAM) and gyroscopic rate sensors.

Since the system is modeled using unit quaternions and the standard Unscented Kalman Filter uses operations of sums, this filter can not be applied straightforwardly to the system considered here. We, then, use rotation vectors, a \(\mathbb{R}^3\) parameterization of the unit quaternion set (the sphere of dimension 3), within the algorithm of the UKF. In numerical simulations, the new filter is shown to provide good estimation and also to outperform the Multiplicative Extended Kalman Filter.

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